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Comments on "Conversions Between *S*, *Z*, *Y*, *h*, *ABCD*, and *T* Parameters which are Valid for Complex Source and Load Impedances"

Roger B. Marks and Dylan F. Williams

In his recent paper,[1] Frickey presents formulas for conversions between various network matrices. Four of these matrices (*Z*, *Y*, *h*, and *ABCD*) relate voltages and currents at the ports; the other two (*S* and *T*) relate wave quantities. These relationships depend on the definitions of the waves themselves in terms of voltage and current. Frickey's results are based on an unconventional definition of the waves, whose resulting properties are unfamiliar to most mircrowave engineers. As a result, application of his formulas can easily lead to catastrophic errors.

The scattering and transmission matrices of classical microwave circuit theory (e.g., [1]–[3]) relate the complex amplitudes of the counterpropagating traveling waves in a transmission line. These modal waves are solutions of Maxwell's equations whose dependence on the axial coordinate z is $e^{\pm\gamma z}$, where γ is the propagation constant. Ratios of the traveling wave amplitudes can be measured by classical slotted line techniques or with a network analyzer using a thru-reflect-line (TRL) calibration [4].

The classical circuit theory also allows the possibility of renormalizing the traveling waves by introducing a reference impedance Z_{ref} that may differ from the characteristic impedance Z_o. The resulting quantities form the basis of a renormalized scattering matrix. For instance, the renormalized reflection coefficient (one-port scattering matrix) Γ of a load of impedance Z_{load}, using a reference impedance Z_{ref}, is simply

$$\Gamma = \frac{Z_{\mathrm{load}} - Z_{\mathrm{ref}}}{Z_{\mathrm{load}} + Z_{\mathrm{ref}}} = \frac{Z_{\mathrm{load}}/Z_{\mathrm{ref}} - 1}{Z_{\mathrm{load}}/Z_{\mathrm{ref}} + 1}. \quad (1)$$

This familiar form is the basis of the Smith Chart, which provides a convenient graphical method of transforming between the reflection coefficient and the normalized load impedance $Z_{\mathrm{load}}/Z_{\mathrm{ref}}$, which, as shown by (1), uniquely determines Γ.

Instead of traveling waves, Frickey [1] makes use of parameters that Youla [5] defines and calls "waves"; a form of these parameters known as "power waves" has previously been applied to microwave circuits [6]. In spite of the terminology, Youla's parameters have little in common with waves. For instance, they do not depend exponentially or even monotonically on z [4]. Furthermore, the properties of Youla's parameters differ fundamentally from those of the renormalized traveling waves. For example, Youla's reflection coefficient $\hat{\Gamma}$ is

$$\hat{\Gamma} = \frac{Z_{\mathrm{load}} - Z^*_{\mathrm{ref}}}{Z_{\mathrm{load}} + Z_{\mathrm{ref}}} = \frac{Z_{\mathrm{load}}/Z_{\mathrm{ref}} - Z^*_{\mathrm{ref}}/Z_{\mathrm{ref}}}{Z_{\mathrm{load}}/Z_{\mathrm{ref}} + 1}. \quad (2)$$

Since (1) does not apply, the Smith Chart is *inapplicable* to Youla's parameters. In fact, $\hat{\Gamma}$ is not even uniquely determined by $Z_{\mathrm{load}}/Z_{\mathrm{ref}}$, as is Γ. As an illustration, the renormalized reflection coefficient of a short circuit ($Z_{\mathrm{load}} = 0$) is always $\Gamma = -1$, regardless of reference impedance Z_{ref}. In contrast, (2) shows that Youla's reflection coefficient of a short is equal *not* to -1 but to $-Z^*_{\mathrm{ref}}/Z_{\mathrm{ref}}$, which has magnitude 1 but is not generally real.

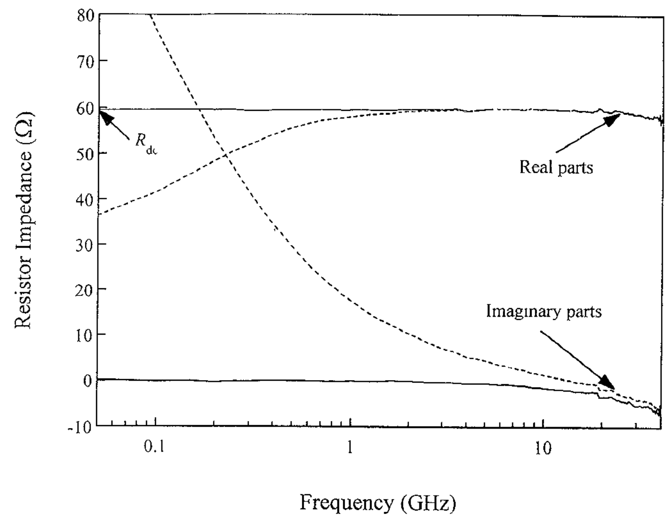

Fig. 1. The impedance of a small lumped resistor calculated, using $Z_{\mathrm{ref}} = Z_o$, from scattering parameters measured by the multiline TRL calibration. The solid curves are calculated from (1), the dashed curves from (2).

No microwave instrumentation or calibration known to us measures Youla's waves [4]. Thus, the equations of the above paper cannot be used to determine impedance parameters from measured scattering parameters. To illustrate, we used the multiline TRL calibration [7] to measure the scattering parameters of a small lumped resistor (with measured dc resistance $R_{\mathrm{dc}} = 59.3\ \Omega$) embedded in a coplanar waveguide. We measured the characteristic impedance Z_o of the transmission line using the technique of [8] and [9]. In applying (1) and (2), we made use of the fact that $Z_{\mathrm{ref}} = Z_o$, a condition which, as is well known, is mandated by the TRL calibration [4], [10]. We determined the resistor impedance Z_{load} first using (1). The result, shown in the solid curves of Fig. 1, closely tracks the resistor's anticipated behavior: the real part is approximately 59 Ω, and the imaginary part is small, approaching zero approximately linearly at low frequencies. When we instead used (2) to calculate Z_{load}, under the assumption that the measured reflection coefficient is actually $\hat{\Gamma}$, we found an anomalous result (dashed curves of Fig. 1).

Due to the unconventional definition of Youla's waves, they can easily lead to erroneous results. For example, consider the simple flow graph of Fig. 2. When the two devices are joined at a reflectionless connector, we generally assume that, as long as the reference impedances at adjoining ports are identical, we can model the circuit by using the simple boundary conditions

$$b_3 = a_2 \quad (3)$$

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[1] D. A. Frickey, *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 205–211, Feb. 1994.

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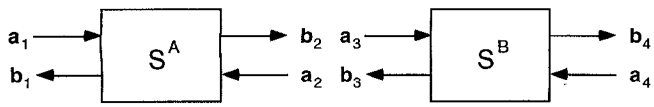

Fig. 2. Signal flow graph of cascaded two-ports.

and

$$a_3 = b_2. \quad (4)$$

In the classical waveguide circuit theory, these conditions arise directly from the continuity of the voltage and current. They are so fundamental as to be intuitive, and they form the basis of signal flow graph analysis and indeed of circuit modeling in general. However, when a and b are Youla's waves, the boundary conditions (3) and (4) do *not* apply. In other words, Youla's waves are not subject to signal flow graph analysis. A corollary is that Frickey's defined transmission matrices, formed from the scattering parameters using his Table VI, do not *function* as transmission matrices. In other words, let us denote the transmission matrix of A by T^A, that of B by T^B, and that of the circuit AB by T^{AB}. A functional transmission matrix must satisfy the condition that $T^A T^B = T^{AB}$. However, algebraic manipulation of Frickey's expressions for the transmission matrix in terms of voltage-current parameters confirms that, for his definitions

$$T^A T^B \neq T^{AB}. \quad (5)$$

Equality in (5) holds true only when the reference impedances on adjoining ports are complex conjugates, a restriction with numerous negative implications. This result of the above paper demonstrates that the counterintuitive nature of Youla's waves can easily lead to serious errors.

In the above paper, Frickey compares his results to those of a commercial simulator. From that comparison, it appears that the simulator also defines scattering parameters in terms of Youla's parameters. This suggests caution in the use of scattering parameters based on a complex reference impedance.

An alternative to Youla's theory is the general waveguide circuit theory of [4], which preserves the essential features of the classical theory while allowing for complex characteristic and reference impedances.

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Reply to Comments on "Conversions Between *S*, *Z*, *Y*, *h*, *ABCD*, and *T* Parameters which are Valid for Complex Source and Load Impedances"

D. A. Frickey

I would like to thank Mr. Marks and Mr. Williams for pointing out the error in using the definition of a_j and b_j in the above paper[1] as I was unaware of the implications involved. Also, I would like to thank the authors for bringing to my attention their work in [1].

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Comments on "An Equivalent Transformation for the Mixed Lumped Lossless Two-port and Distributed Transmission Line"

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Stimulated by previous articles [1]–[4] by the authors of the above paper,[1] we have done related research. In doing so, we have found additional results and synthesis applications ([5], parts also in [6]) that we would like to communicate here briefly.

In [6] and (more conveniently in [5]) we gave formulas for the transformation of the D section with l'Hospital's rule already incorporated, so that no indefinite expressions such as 0/0 (cf. p. 277, text between (80) and (81)) occur. According formulas for the other sections are also given in [5], [6]. These formulas seem to be more suited for the use in the synthesis applications described below.

The equivalent transformation treated in the Theorem in Section V of the above paper, which we in accordance to the idiomatic usage in [1], [2] and due to [7] called extended Levy transformation, can also be performed numerically. This can be done by solving a system of ordinary differential equations, where the line length l is the independent variable and the coefficients of the numerators of the lumped lossless two-port chain matrix elements are the functions to be determined. Reference [5] contains some additional theorems on the asymptotic behavior of this transformation for $l \to \infty$.

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